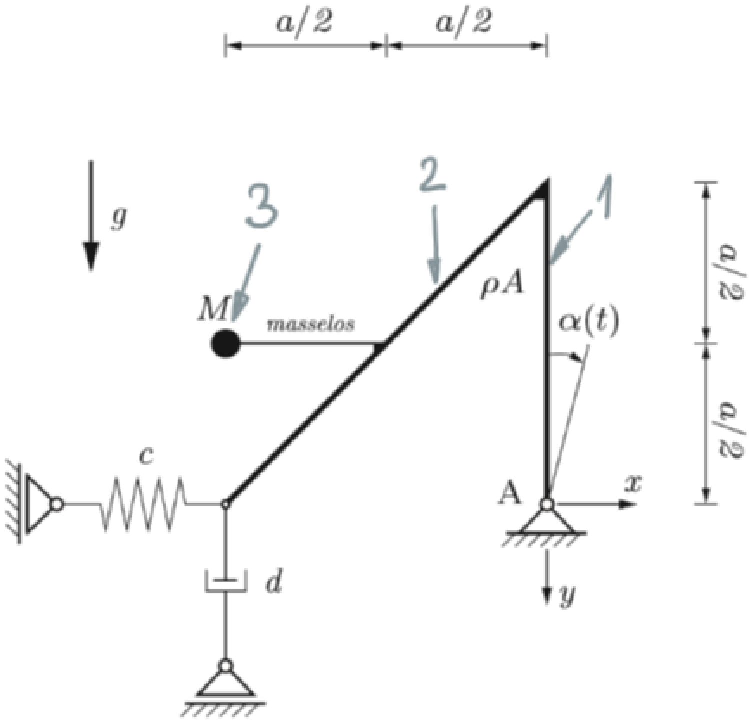
Gegeben:

- System mit einem Freiheitsgrad $\alpha(t)$ mit Abmessungen gemäß Bild
- Masse M, Masse pro Länge ρA, Federkonstante c und Dämpferkonstante d
- Feder ist für $\alpha = 0$ entspannt!



1. Massenträgheitsmoment:

$m = \rho \cdot A \cdot a$

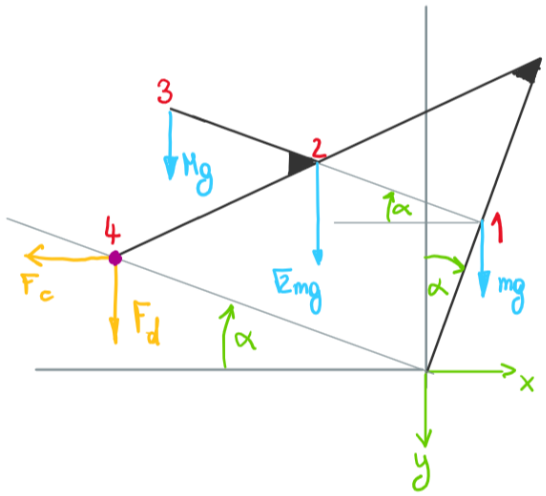
$$I_{(A)1} = \frac{ma^2}{12} + m\left(\frac{a}{2}\right)^2 = \frac{ma^2}{3}$$

$$I_{(A)2} = \frac{\sqrt{2}\, m\, (\sqrt{2}\, a)^2}{12} + \sqrt{2}\, m\left(\left(\frac{a}{2}\right)^2 + \left(\frac{a}{2}\right)^2\right) = \frac{2\sqrt{2}\, ma^2}{3}$$

$$I_{(A)3} = M\left(a^2 + \left(\frac{a}{2}\right)^2\right) = \frac{5Ma^2}{4}$$

$$I_{(A)} = \frac{ma^2}{3} + \frac{2\sqrt{2}\, ma^2}{3} + \frac{5Ma^2}{4}$$

2. Freimachen:



$F_c = c \cdot \Delta x \quad , \Delta x = x_0 - x = a - a\cos\alpha$

$= c \cdot a \cdot (1 - \cos\alpha)$

$F_d = d \cdot \Delta\dot{y} \quad , \Delta y = a \cdot \sin\alpha$

$\Delta\dot{y} = \frac{d}{dt}(\Delta y) = \dot{\alpha} \cdot a \cdot \cos\alpha$

$F_d = d \cdot a \cdot \dot{\alpha} \cdot \cos\alpha$

Drehimpulssatz:

$$I_{(A)} \cdot \ddot{\alpha} = -F_c\, a \cdot \sin\alpha - F_d\, a \cdot \cos\alpha + mg\frac{a}{2}\sin\alpha - \sqrt{2}\,mg\frac{a}{2}(\cos\alpha - \sin\alpha) - Mg\,a\left(\cos\alpha - \frac{1}{2}\sin\alpha\right)$$

$$I_{(A)}\ddot{\alpha} + d a^2 \dot{\alpha}\cos^2\alpha + c a^2(1-\cos\alpha)\sin\alpha = mg\frac{a}{2}\sin\alpha - \sqrt{2}\,mg\frac{a}{2}(\cos\alpha - \sin\alpha) - Mg\,a\left(\cos\alpha - \frac{1}{2}\sin\alpha\right)$$

... nichtlineare BWGL

d'Alembertsches Prinzip:

$$\underline{r}_1 = \frac{a}{2}\begin{pmatrix} \sin\alpha \\ -\cos\alpha \end{pmatrix} \quad ; \quad \delta\underline{r}_1 = \delta\alpha\,\hat{\underline{r}}_1 = \frac{a}{2}\,\delta\alpha\begin{pmatrix} \cos\alpha \\ \sin\alpha \end{pmatrix}$$

$$\underline{r}_2 = \frac{a}{2}\begin{pmatrix} -(\cos\alpha - \sin\alpha) \\ -(\cos\alpha + \sin\alpha) \end{pmatrix} \quad ; \quad \delta\underline{r}_2 = \frac{a}{2}\,\delta\alpha\begin{pmatrix} \cos\alpha + \sin\alpha \\ -(\cos\alpha - \sin\alpha) \end{pmatrix}$$

$$\underline{r}_3 = \frac{a}{2}\begin{pmatrix} -(2\cos\alpha - \sin\alpha) \\ -(\cos\alpha + 2\sin\alpha) \end{pmatrix} \quad ; \quad \delta\underline{r}_3 = \frac{a}{2}\,\delta\alpha\begin{pmatrix} \cos\alpha + 2\sin\alpha \\ -(2\cos\alpha - \sin\alpha) \end{pmatrix}$$

$$\underline{r}_4 = a\begin{pmatrix} -\cos\alpha \\ -\sin\alpha \end{pmatrix} \quad ; \quad \delta\underline{r}_4 = a\,\delta\alpha\begin{pmatrix} \sin\alpha \\ -\cos\alpha \end{pmatrix}$$

$\delta A^{(a)} = \delta A^{(tr)}$ (Skriptum: $\delta A^{(a)} + \delta A^{(tr)} = 0$)

Trägheitskräfte:

$$\delta A^{(tr)} = -\frac{dJ}{dt}\cancel{\delta r_A} \overset{-}{+} \left(\frac{d}{dt}D_{(A)} + m\, r_{MA} \times \cancel{a_A}\right)\delta\alpha = \frac{d}{dt}\left(D_{(A)}\right)\delta\alpha = \frac{d}{dt}\left(I_{(A)}\dot{\alpha}\right)\delta\alpha = I_{(A)}\ddot{\alpha}\,\delta\alpha$$

$A \hat{=}$ Festlager

virtuelle Arbeit der äußeren Kräfte:

$$\delta A^{(a)} = \begin{bmatrix} 0 \\ mg \end{bmatrix}\delta r_1 + \begin{bmatrix} 0 \\ \sqrt{2}mg \end{bmatrix}\delta r_2 + \begin{bmatrix} 0 \\ Mg \end{bmatrix}\delta r_3 + \begin{bmatrix} -F_c \\ F_d \end{bmatrix}\delta r_4$$

$$\Rightarrow \delta A^{(a)} = mg\frac{a}{2}\sin\alpha\,\delta\alpha - \sqrt{2}mg\frac{a}{2}\delta\alpha\left(\cos\alpha - \sin\alpha\right) - Mg\frac{a}{2}\delta\alpha\left(2\cos\alpha - \sin\alpha\right)$$
$$-ca^2(1-\cos\alpha)\sin\alpha\,\delta\alpha - da^2\dot{\alpha}\cos^2\alpha\,\delta\alpha$$

$$\Rightarrow \left(I_{(A)}\ddot{\alpha}\right)\cancel{\delta\alpha} = \left(-ca^2(1-\cos\alpha)\sin\alpha - da^2\dot{\alpha}\cos^2\alpha + mg\frac{a}{2}\sin\alpha - \sqrt{2}mg\frac{a}{2}(\cos\alpha - \sin\alpha) - Mga\left(\cos\alpha - \frac{1}{2}\sin\alpha\right)\right)\cancel{\delta\alpha}$$

... nichtlineare BWGL

Linearisierung:

stat. Ruhelage: $\ddot{\alpha}_S = \dot{\alpha}_S = 0$

$$\Rightarrow 0 = -ca^2(1-\cos\alpha_S)\sin\alpha_S + mg\frac{a}{2}\sin\alpha_S - \sqrt{2}mg\frac{a}{2}(\cos\alpha_S - \sin\alpha_S) - Mga\left(\cos\alpha_S - \frac{1}{2}\sin\alpha_S\right)$$

- $\alpha = \alpha_S + \varepsilon$

Terme höherer Ordnung

$$f(\alpha,\dot{\alpha},\ddot{\alpha}) = \cancelto{0}{f(\alpha_S,0,0)} + \left.\frac{\partial f}{\partial\ddot{\alpha}}\right|_{RL}\ddot{\varepsilon} + \left.\frac{\partial f}{\partial\dot{\alpha}}\right|_{RL}\dot{\varepsilon} + \left.\frac{\partial f}{\partial\alpha}\right|_{RL}\varepsilon + \ldots$$

$$f(\alpha,\dot{\alpha},\ddot{\alpha}) = I_{(A)}\ddot{\alpha} + da^2\dot{\alpha}\cos^2\alpha + ca^2(1-\cos\alpha)\sin\alpha - mg\frac{a}{2}\sin\alpha + \sqrt{2}mg\frac{a}{2}(\cos\alpha - \sin\alpha) + Mga\left(\cos\alpha - \frac{1}{2}\sin\alpha\right)$$

$$\left.\frac{\partial f}{\partial\ddot{\alpha}}\right|_{RL} = I_{(A)}\quad,\quad \left.\frac{\partial f}{\partial\dot{\alpha}}\right|_{RL} = da^2\cos^2\alpha_S$$

$$\left.\frac{\partial f}{\partial\alpha}\right|_{RL} = ca^2\left((1-\cos\alpha_S)\cos\alpha_S + \sin^2\alpha_S\right) - mg\frac{a}{2}\cos\alpha_S - \sqrt{2}mg\frac{a}{2}(\sin\alpha_S + \cos\alpha_S) - Mga\left(\sin\alpha_S + \frac{1}{2}\cos\alpha_S\right)$$

$$I_{(A)}\ddot{\varepsilon} = -da^2\cos^2\alpha_S\,\dot{\varepsilon} - ca^2\left(\cos\alpha_S - \cos^2\alpha_S + \sin^2\alpha_S\right)\cdot\varepsilon + \left[mg\frac{a}{2}\cos\alpha_S + \sqrt{2}mg\frac{a}{2}(\sin\alpha_S + \cos\alpha_S) + Mga\left(\sin\alpha_S + \frac{1}{2}\cos\alpha_S\right)\right]\cdot\varepsilon$$

... linearisierte BWGL